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Are lepton flavor mixings in the democratic mass matrix stable against quantum corrections?

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Abstract

We investigate whether the lepton flavor mixing angles in the so-called democratic type of mass matrix are stable against quantum corrections or not in the minimal supersymmetric standard model with dimension five operator which induces neutrino mass matrix. By taking simple breaking patterns of $S_{3L} \times S_{3R}$ or $O(3)_L \times O(3)_R$ flavor symmetries and the scale where democratic textures are induced as $O(10^{13})$ GeV, we find that the stability of the lepton flavor mixing angles in the democratic type of mass matrix against quantum corrections depends on the solar neutrino solutions. The maximal flavor mixing of the vacuum oscillation solution is spoiled by the quantum corrections in the experimental allowed region of $\tan\beta$. The large angle MSW solution is spoiled by the quantum corrections in the region of $\tan\beta > 10$. The condition of $\tan\beta \leq 10$ is needed in order to obtain the suitable mass squared difference of the small angle MSW solution. These strong constraints must be regarded for the model building of the democratic type of mass matrix.

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Recent neutrino oscillation experiments suggest the strong evidences of tiny neutrino masses and lepton flavor mixings [1, 2, 3, 4]. Studies of the lepton flavor mixing matrix, which is so-called Maki-Nakagawa-Sakata (MNS) matrix[5], will give us important cues of the physics beyond the standard model. One important study is finding the suitable texture of quark and lepton mass matrices in order to search the flavor symmetry existing behind. The democratic type of mass matrix[6] is one of the most interesting candidate of the texture of quark and lepton mass matrices, since it can naturally explain the reason why only masses of third generation particles are large comparing to those of other generations. This type of mass matrix can be derived by the flavor symmetries of $S_{3L} \times S_{3R}$[7, 8, 9] or $O(3)_L \times O(3)_R$[10]. As for the neutrino sector, it has been said that the democratic type of mass matrix can induce the suitable solutions of the atmospheric and the solar neutrino problems [7, 8, 9, 10]. However, are lepton flavor mixing angles in the democratic type of mass matrix stable against quantum corrections?

In this paper, we investigate whether the lepton flavor mixing angles in the democratic type of mass matrix are stable against quantum corrections or not in the minimal supersymmetric standard model with the dimension five operator which induces the neutrino Majorana mass matrix. The superpotential of the lepton-Higgs interactions is given by

$$\mathcal{W} = y^{\mathrm{e}}_{ij}(H_d L_i)E_j - \frac{1}{2}\kappa_{ij}(H_u L_i)(H_u L_j)\,, \tag{1}$$

where κ is the coefficient of the dimension five operator, and the indices i, j $(= 1 \sim 3)$ stand for the generation number. L_i and E_i are chiral super-fields of i-th generation lepton doublet and right-handed charged-lepton, respectively. H_u (H_d) is the Higgs doublet which gives Dirac masses to the up- (down-) type fermions. We will show the renormalization group equation (RGE) analyses of the lepton flavor mixing angles [11, 12, 13].

We take the simple breaking patterns of $S_{3L} \times S_{3R}$ or $O(3)_L \times O(3)_R$ symmetries, and the scale where democratic textures are induced as $O(10^{13})$ GeV. Under the above conditions, we find that the stability of the lepton flavor mixing angles in the democratic type of mass matrix against quantum corrections depends on the solar neutrino solutions. The maximal flavor mixing of the vacuum oscillation (VO) solution[14] is spoiled by the quantum corrections in the experimental allowed region of $\tan\beta$. The value of $\tan\beta$ is the ratio between the vacuum expectation values (VEVs) of the Higgs particles. The large angle MSW (MSW-L) solution[15] is spoiled by the quantum corrections in the region of $\tan\beta > 10$. On the other hand, the condition of $\tan\beta \leq 10$ is needed in order to obtain the suitable mass squared difference of the small angle MSW solution (MSW-S)[15]. These strong constraints must be regarded for the model building of the democratic type of mass matrix.

At first, we discuss the democratic mass matrix, which is based on $S_{3L} \times S_{3R}$ or $O(3)_L \times O(3)_R$ flavor symmetries. In the democratic type of mass matrix, the charged-lepton mass matrix is given by

$$M_l = \frac{c_l}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + M_l^{(c)}\,, \tag{2}$$

where $M_l^{(c)}$ includes flavor symmetry breaking masses, which must be introduced to obtain the suitable values of m_e and m_μ. The matrix M_l is diagonalized by the unitary matrix $V_l = FL$ from the side of left-handed fields, where

$$F = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix}. \tag{3}$$

Since we do not know the definite structure of $M_l^{(c)}$, we can not determine the explicit form of the unitary matrix L. Here we assume that off diagonal elements of L are small as $L_{ij} \ll 1$ $(i \neq j)$ from the analogy of the quark sector [7, 8, 9, 10]. Thus, we obtain the relation of $V_l \simeq F$, which is used in the following discussions*.

The neutrino mass matrix is given by

$$M_\nu = c_\nu \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right\} + M_\nu^{(b)}\,, \tag{4}$$

since the neutrinos are Majorana particles. In Eq.(4), c_ν and r can be taken as real and non-negative parameters, and we neglect CP phases, for simplicity. The mass matrix $M_\nu^{(b)}$ breaks the flavor symmetries, which must be introduced in order to obtain the suitable mass squared differences and mixing angles of neutrinos. There are following two simple breaking patterns according to the solar neutrino solutions.

(i): The simplest example of $M_\nu^{(b)}$ for the MSW-L and the VO solutions is to introduce two real and non-negative parameters ϵ and δ $(1 \gg \delta \gg \epsilon)$ in (2,2) and (3,3) elements in $M_\nu^{(b)}$, where the neutrino mass matrix $M_\nu^{(i)}$ is given by

$$M_\nu^{(i)} = c_\nu \begin{pmatrix} 1+r & r & r \\ r & 1+r+\epsilon & r \\ r & r & 1+r+\delta \end{pmatrix}. \tag{5}$$

(ii): The simplest example of $M_\nu^{(b)}$ for the MSW-S solution is to introduce two real and non-negative parameters ϵ and δ $(1 \gg \delta \gg \epsilon)$ in (1,2), (2,1), and (3,3) elements in $M_\nu^{(b)}$,

*We will revive L_{ij} in the MSW-S solution later.

where the neutrino mass matrix $M_\nu^{(ii)}$ is given by

$$M_\nu^{(ii)} = c_\nu \begin{pmatrix} 1+r & r+\epsilon & r \\ r+\epsilon & 1+r & r \\ r & r & 1+r+\delta \end{pmatrix} . \tag{6}$$

When $r \gg \epsilon$, δ, the unitary matrix U_ν, which diagonalizes M_ν, becomes F in both cases of (i) and (ii). In this case the MNS matrix approaches to the unit matrix as

$$V_{MNS} = V_l^\dagger U_\nu \simeq L^\dagger F^\dagger F \simeq L^\dagger, \tag{7}$$

which does not have any large mixing angles. Therefore the magnitude of r must be smaller than ϵ, δ in order to obtain the large flavor mixing of the atmospheric neutrino solution†. Thus, in the democratic type of mass matrix, three neutrinos are degenerate with the same signs, where the RGE effects cannot be negligible as shown in case (c4) in Ref.[13]. This is the reason why we need RGE analyses for the democratic type of mass matrix.
Under the condition of $r \ll \epsilon, \delta$, both simple breaking patterns of (i) and (ii) induce the large mixing angle of $\sin^2 2\theta_{23} \simeq 8/9$ which is suitable for the atmospheric neutrino solution[2, 3], and negligibly small mixing between the first and the third generations as $\sin^2 2\theta_{13} \simeq 0$ which is consistent with the CHOOZ experiment [4]. Case (i) induces the maximal mixing between the first and the second generations for the solar neutrino solution as $\sin^2 2\theta_{12} \simeq 1$[7, 8, 9, 10]. On the other hand, case (ii) induces the small mixing between the first and the second generations, since the maximal mixing angles induced from both M_l and M_ν are canceled with each other[8].

Now let us estimate the quantum corrections of the MNS matrix in the democratic type of mass matrix. We take the diagonal base of the charged-lepton mass matrix at the high energy scale m_h, where the democratic textures are induced, for the RGE analysis. In this base the neutrino mass matrix in Eqs.(5) and (6) are written by $V_l^\dagger M_\nu(m_h) V_l \simeq F^T M_\nu(m_h) F$, where we use the approximation of $V_l \simeq F$. Then, the neutrino mass matrix at m_Z scale is given by

$$M_\nu\left(m_Z\right) = \frac{M_\nu\left(m_Z\right)_{33}}{M_\nu\left(m_h\right)_{33}}\, R_G F^T M_\nu(m_h) F R_G\,, \tag{8}$$

†We do not consider the case of $r = -2/3$[16] which gives degenerate neutrinos, where one of mass eigen-values has a opposite sign from others. It is because the simple symmetry breaking patterns such as (i) and (ii) can not induce the large lepton flavor mixing as shown above. In the case of $r \gg \epsilon, \delta$, the flavor symmetry breaking textures must be complicated in order to solve both the solar and the atmospheric neutrino problems.

where the matrix R_G shows the renormalization effects, which is defined as

$$R_G \equiv \begin{pmatrix} 1+\eta & 0 & 0 \\ 0 & 1+\eta & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{9}$$

The small parameter η is given by

$$\begin{aligned} \eta &\simeq 1 - \exp\left(-\frac{1}{16\pi^2}\int_{\ln(m_Z)}^{\ln(m_h)} y_\tau^2 dt\right), \\ &\simeq \frac{1}{8\pi^2}\frac{m_\tau^2}{v^2}\left(1+\tan^2\beta\right)\ln\left(\frac{m_h}{m_Z}\right) , \end{aligned} \tag{10}$$

where y_τ is the Yukawa coupling of τ and $v^2 \equiv \langle H_u\rangle^2 + \langle H_d\rangle^2$. We neglect the Yukawa couplings of e and μ in Eq.(9), since those contributions to the RGE are negligibly small comparing to that of τ[13]. Therefore the first and the second generations receive the same RGE corrections as in Eq.(9). Now let us check whether the mixing angles receive significant changes by the quantum corrections or not in both cases of (i) and (ii).

In the base of charged-lepton democratic mass matrix, $M_\nu(m_Z)$ in case (i) is written as

$$M_\nu^{(i)}(m_Z) = F R_G F^T M_\nu^{(i)}(m_h) F R_G F^T , \tag{11}$$

$$\simeq \bar{c_\nu}\begin{pmatrix} 1+\bar{r} & \bar{r} & \bar{r} \\ \bar{r} & 1+\bar{r}+\epsilon & \bar{r} \\ \bar{r} & \bar{r} & 1+\bar{r}+\delta \end{pmatrix} + 2\eta\bar{c_\nu}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{12}$$

where

$$\bar{r} \equiv r - \frac{2}{3}\eta, \qquad \bar{c_\nu} \equiv \frac{M_\nu\left(m_Z\right)_{33}}{M_\nu\left(m_h\right)_{33}} c_\nu . \tag{13}$$

Here we neglect the small parameters of order ϵ^2, $\epsilon\eta$, and $\epsilon\delta$. Equation (12) means that the MNS matrix at m_Z scale is obtained only by using $\bar{r}$ instead of r in Eq.(5). This had been already shown in Ref.[9]. Therefore all we have to do for the RGE analyses of the mixing angles is to trace the change of $\bar{r}$. Here we remind that the magnitude of η is completely determined by the value of $\tan\beta$ and the scale of m_h[13], then $\bar{r}$ is determined by Eq.(13).

Now let us show how the MNS matrix changes as the change of $\bar{r}$.

(i-a): $1 \gg \delta \gg \epsilon \gg |\bar{r}|$

Neglecting the second order of small parameters of $\delta, \bar{r}$, and ϵ, mass eigen-values of $M_\nu^{(i)}(m_Z)$ are give by

$$\bar{c_\nu}(1+\bar{r}+2\eta), \qquad \bar{c_\nu}(1+\bar{r}+\epsilon+2\eta), \qquad \bar{c_\nu}(1+\bar{r}+\delta+2\eta) . \tag{14}$$

Then the unitary matrix U_ν becomes

$$U_\nu \simeq \begin{pmatrix} 1 & \frac{\bar{r}}{\epsilon} & \frac{\bar{r}}{\delta} \\ -\frac{\bar{r}}{\epsilon} & 1 & \frac{\bar{r}}{\delta} \\ -\frac{\bar{r}}{\delta} & -\frac{\bar{r}}{\delta} & 1 \end{pmatrix}, \tag{15}$$

which induces the MNS matrix V_{MNS} as

$$V_{MNS} \simeq F^T U_\nu = \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1+\frac{\bar{r}}{\epsilon}\right) & -\frac{1}{\sqrt{2}}\left(1-\frac{\bar{r}}{\epsilon}\right) & 0 \\ \frac{1}{\sqrt{6}}\left(1+2\frac{\bar{r}}{\delta}-\frac{\bar{r}}{\epsilon}\right) & \frac{1}{\sqrt{6}}\left(1+2\frac{\bar{r}}{\delta}+\frac{\bar{r}}{\epsilon}\right) & -\sqrt{\frac{2}{3}}\left(1-\frac{\bar{r}}{\delta}\right) \\ \frac{1}{\sqrt{3}}\left(1-\frac{\bar{r}}{\delta}-\frac{\bar{r}}{\epsilon}\right) & \frac{1}{\sqrt{3}}\left(1-\frac{\bar{r}}{\delta}+\frac{\bar{r}}{\epsilon}\right) & \frac{1}{\sqrt{3}}\left(1+2\frac{\bar{r}}{\delta}\right) \end{pmatrix}. \tag{16}$$

Thus the mixing angles are given by

$$\sin^2 2\theta_{12} = 1 - 4(\frac{\bar{r}}{\epsilon})^2, \qquad \sin^2 2\theta_{13} = 0, \qquad \sin^2 2\theta_{23} = \frac{8}{9}\left(1+2\frac{\bar{r}}{\delta} - 9(\frac{\bar{r}}{\delta})^2\right). \tag{17}$$

This means all mixing angles are not changed by the quantum corrections in the region of $1 \gg \delta \gg \epsilon \gg |\bar{r}|$. Equation (14) shows that $\Delta m_{12}^2 \simeq 2\bar{c_\nu}^2\epsilon$ and $\Delta m_{23}^2 \simeq 2\bar{c_\nu}^2\delta$. In order for the symmetry breaking parameter δ to be smaller than symmetric terms of order one, it must be that $\delta \leq O(0.1)$. On the other hand, neutrino-less $\beta\beta$-decay experiments suggest $\bar{c_\nu} \leq O(0.1)$ eV. Then, $\bar{c_\nu} = O(0.1)$ eV and $\delta = O(0.1)$ are obtained from the experimental results of $\Delta m_{\rm ATM}^2 \simeq 10^{-3}$ eV^2. This means that $\epsilon = O(10^{-3})$ for the MSW-L solution, and $\epsilon = O(10^{-8})$ for the VO solution. Then, the region of $\epsilon \gg \bar{r}$ corresponds to $\tan\beta < 10$[‡] for the MSW-L solution and $\tan\beta \ll 1$ for the VO solution[13]. Since the region of $\tan\beta \ll 1$ is excluded by the Higgs search experiments[17], we can conclude the maximal mixing of the VO solution in the democratic type of mass matrix, discussed in Refs.[7, 8], is completely spoiled by the quantum corrections[§]. For the MSW-L solution, discussed in Refs.[9, 10], the sufficient condition of $\tan\beta < 10$ must be satisfied.

<u>(i-b): $1 \gg \delta \gg |\bar{r}| \gg \epsilon$</u>

Neglecting the second order of small parameters of $\delta, \bar{r}$, and ϵ, mass eigen-values of $M_\nu^{(i)}(m_Z)$ are give by

$$\bar{c_\nu}(1+\frac{1}{2}\epsilon+2\eta), \qquad \bar{c_\nu}(1+2\bar{r}+\frac{1}{2}\epsilon+2\eta), \qquad \bar{c_\nu}(1+\bar{r}+\delta+2\eta)\,. \tag{18}$$

‡The higher the scale of m_h becomes, the smaller the value of $\tan\beta$ must be in order for the maximal flavor mixing not to be destroyed by the quantum corrections.

§The maximal mixing of the VO solution is not spoiled by the quantum corrections (even in $\tan\beta = 3$), if $m_h \leq O(1)$ TeV. However, such a low energy scale of m_h is not suitable from the view point of model building.

The unitary matrix U_ν becomes

$$U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{2}}\left(1-\frac{1}{4}\frac{\epsilon}{\bar{r}}\right) & \frac{\bar{r}}{\delta} \\ -\frac{1}{\sqrt{2}}\left(1-\frac{1}{4}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\frac{\epsilon}{\bar{r}}\right) & \frac{\bar{r}}{\delta} \\ -\frac{1}{2\sqrt{2}}\frac{\epsilon}{\delta} & -\sqrt{2}\frac{\bar{r}}{\delta} & 1 \end{pmatrix}, \tag{19}$$

which induces the MNS matrix as

$$V_{MNS} \simeq F^T U_\nu = \begin{pmatrix} 1 & -\frac{1}{4}\frac{\epsilon}{\bar{r}} & 0 \\ \frac{1}{2\sqrt{3}}\frac{\bar{r}}{\delta} & \frac{1}{\sqrt{3}}\left(1+\frac{1}{2}\frac{\bar{r}}{\delta}\right) & -\sqrt{\frac{2}{3}}\left(1-\frac{\bar{r}}{\delta}\right) \\ -\frac{1}{2\sqrt{6}}\left(\frac{\epsilon}{\delta}-\frac{\epsilon}{\bar{r}}\right) & \sqrt{\frac{2}{3}}\left(1-\frac{\bar{r}}{\delta}\right) & \frac{1}{\sqrt{3}}\left(1+2\frac{\bar{r}}{\delta}\right) \end{pmatrix}. \tag{20}$$

Then the mixing angles are given by

$$\sin^2 2\theta_{12} = \frac{1}{4}\left(\frac{\epsilon}{\bar{r}}\right)^2, \quad \sin^2 2\theta_{13} = 0, \quad \sin^2 2\theta_{23} = \frac{8}{9}\left(1+2\frac{\bar{r}}{\delta}-9(\frac{\bar{r}}{\delta})^2\right). \tag{21}$$

This means that maximal mixings of all solar solutions in the democratic type of mass matrix of Eq.(12) are spoiled by the quantum corrections in the region of $1 \gg \delta \gg |\bar{r}| \gg \epsilon$, although the mixings between the first and the third generations, and between the second and the third generations are stable against quantum corrections .

<u>(i-c): $1 \gg \ |\bar{r}| \gg \delta \gg \epsilon$</u>

Neglecting the second order of small parameters of $\delta, \bar{r}$, and ϵ, mass eigen-values of $M_\nu^{(i)}(m_Z)$ are give by

$$\bar{c_\nu}(1+\frac{1}{2}\epsilon+2\eta), \quad \bar{c_\nu}(1+\frac{2}{3}\delta+\frac{1}{6}\epsilon+2\eta), \quad \bar{c_\nu}(1+3\bar{r}+\frac{1}{3}\delta+\frac{1}{3}\epsilon+2\eta). \tag{22}$$

In this case U_ν becomes

$$U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\left(1+\frac{1}{4}\frac{\epsilon}{\delta}+\frac{1}{6}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{6}}\left(1-\frac{3}{4}\frac{\epsilon}{\delta}+\frac{2}{9}\frac{\delta}{\bar{r}}-\frac{1}{9}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}-\frac{1}{9}\frac{\epsilon}{\bar{r}}\right) \\ -\frac{1}{\sqrt{2}}\left(1-\frac{1}{4}\frac{\epsilon}{\delta}-\frac{1}{6}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{6}}\left(1+\frac{3}{4}\frac{\epsilon}{\delta}+\frac{2}{9}\frac{\delta}{\bar{r}}-\frac{1}{9}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}+\frac{2}{9}\frac{\epsilon}{\bar{r}}\right) \\ -\frac{1}{2\sqrt{2}}\left(\frac{\epsilon}{\delta}-\frac{1}{3}\frac{\epsilon}{\bar{r}}\right) & -\sqrt{\frac{2}{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}+\frac{1}{18}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1+\frac{2}{9}\frac{\delta}{\bar{r}}-\frac{1}{9}\frac{\epsilon}{\bar{r}}\right) \end{pmatrix}. \tag{23}$$

Thus, the MNS matrix is given by

$$V_{MNS} \simeq F^T U_\nu = \begin{pmatrix} 1 & -\frac{\sqrt{3}}{4}\frac{\epsilon}{\delta} & -\frac{1}{3\sqrt{6}}\frac{\epsilon}{\bar{r}} \\ \frac{\sqrt{3}}{4}\frac{\epsilon}{\delta} & 1 & -\frac{\sqrt{2}}{9}\left(\frac{\delta}{\bar{r}} - \frac{1}{2}\frac{\epsilon}{\bar{r}}\right) \\ \frac{1}{2\sqrt{6}}\frac{\epsilon}{\bar{r}} & \frac{\sqrt{2}}{9}\left(\frac{\delta}{\bar{r}} - \frac{1}{2}\frac{\epsilon}{\bar{r}}\right) & 1 \end{pmatrix}, \tag{24}$$

which gives the mixing angles as

$$\sin^2 2\theta_{12} = \frac{3}{4}\left(\frac{\epsilon}{\delta}\right)^2, \quad \sin^2 2\theta_{13} = \frac{1}{54}\left(\frac{\epsilon}{\bar{r}}\right)^2, \quad \sin^2 2\theta_{23} = \frac{8}{81}\left(\frac{\delta}{\bar{r}} + \frac{\epsilon}{4\bar{r}}\right)^2. \tag{25}$$

This means that large mixing angles in both the solar and the atmospheric neutrino solutions are spoiled by the quantum corrections in the region of $1 \gg |\bar{r}| \gg \delta \gg \epsilon$. It is because the condition of $|\bar{r}| \gg \delta, \epsilon$ induces $U_\nu \simeq F$, which is just the case of Eq.(7).

The conclusion in case (i) are that (1): the maximal mixing of the VO solution is destroyed by the quantum corrections, and (2): the sufficient condition of $\tan\beta < 10$ must be satisfied for the MSW-L solution.

Next, let us show the case (ii) in the base of charged-lepton democratic mass matrix, where $M_\nu^{(ii)}(m_Z)$ is written by

$$M_\nu^{(ii)}(m_Z) \simeq \bar{c_\nu} \begin{pmatrix} 1+\bar{r} & \bar{r}+\epsilon & \bar{r} \\ \bar{r}+\epsilon & 1+\bar{r} & \bar{r} \\ \bar{r} & \bar{r} & 1+\bar{r}+\delta \end{pmatrix} + 2\eta\bar{c_\nu} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{26}$$

Let us show how the MNS matrix changes according to the change of $\bar{r}$ as in case (i).

<u>(ii-a): $1 \gg \delta \gg \epsilon, |\bar{r}|$</u>

Neglecting the second order of small parameters of $\delta, \bar{r}$, and ϵ, the mass eigen-values of $M_\nu^{(ii)}(m_Z)$ are give by

$$\bar{c_\nu}(1-\epsilon+2\eta), \quad \bar{c_\nu}(1+2\bar{r}+\epsilon+2\eta), \quad \bar{c_\nu}(1+\bar{r}+\delta+2\eta). \tag{27}$$

In this case U_ν becomes

$$U_\nu \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\bar{r}}{\delta} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\bar{r}}{\delta} \\ 0 & -\sqrt{2}\frac{\bar{r}}{\delta} & 1 \end{pmatrix}, \tag{28}$$

which induces the MNS matrix as

$$
\begin{aligned}
V_{MNS} &= L^\dagger F^T U_\nu \qquad (29)\\
&\simeq \begin{pmatrix}
1 & \frac{1}{\sqrt{3}}L_{21}\left(1+2\frac{\bar{r}}{\delta}\right) & -\sqrt{\frac{2}{3}}L_{21}\left(1-\frac{\bar{r}}{\delta}\right) \\
L_{12} & \frac{1}{\sqrt{3}}\left(1+2\frac{\bar{r}}{\delta}\right)+\frac{1}{\sqrt{3}}L_{31}\left(1+\frac{\bar{r}}{\delta}\right) & -\sqrt{\frac{2}{3}}\left(1-\frac{\bar{r}}{\delta}\right)+\frac{1}{\sqrt{3}}L_{32}\left(1+2\frac{\bar{r}}{\delta}\right) \\
L_{13} & \sqrt{\frac{2}{3}}\left(1-\frac{\bar{r}}{\delta}\right)+\frac{1}{\sqrt{3}}L_{23}\left(1+2\frac{\bar{r}}{\delta}\right) & \frac{1}{\sqrt{3}}\left(1+2\frac{\bar{r}}{\delta}\right)-\frac{3}{2}L_{23}\left(1-\frac{\bar{r}}{\delta}\right)
\end{pmatrix},
\end{aligned}
$$

where we revive the small elements of L_{ij} $(i \neq j)$. This shows that the mixing angles are given by

$$
\sin^2 2\theta_{12} = \frac{4}{3}L_{21}^2\left(1+2\frac{\bar{r}}{\delta}\right)^2, \quad \sin^2 2\theta_{13} = \frac{8}{3}L_{21}^2\left(1-\frac{\bar{r}}{\delta}\right)^2, \quad \sin^2 2\theta_{23} = \frac{8}{9}\left(1+2\frac{\bar{r}}{\delta}-9(\frac{\bar{r}}{\delta})^2\right). \tag{30}
$$

This means that all flavor mixings are not spoiled by the quantum corrections in the region of $1 \gg \delta \gg |\bar{r}|, \epsilon$. Equation (27) suggests that $\Delta m_{12}^2 \simeq 4\bar{c_\nu}^2(\bar{r}+\epsilon)$ and $\Delta m_{23}^2 \simeq 2\bar{c_\nu}^2\delta$. Thus, when $|\bar{r}| \geq \epsilon$, the quantum correction is the origin of mass squared difference for the solar neutrino solution. Where we must tune the value of $\tan\beta$ in order to obtain the suitable mass squared difference. The case of $\tan\beta \simeq 10$ induces $\Delta m_{12}^2 \sim 10^{-5}$ eV^2 at m_Z. Therefore the condition of $\tan\beta \leq 10$ must be satisfied in order to obtain the suitable magnitude of mass squared difference, for the MSW-S solution discussed in Ref.[8]. As for the mixings between the first and the third generations, and between the second and the third generations, Eq.(30) shows that they are stable against quantum corrections.

(ii-b): $1 \gg |\bar{r}| \gg \delta \gg \epsilon$

Neglecting the second order of small parameters of $\delta, \bar{r}$, and ϵ, mass eigen-values of $M_\nu^{(ii)}(m_Z)$ are give by

$$
\bar{c_\nu}(1-\epsilon+2\eta), \qquad \bar{c_\nu}(1+\frac{2}{3}\delta+\frac{1}{3}\epsilon+2\eta), \qquad \bar{c_\nu}(1+3\bar{r}+\frac{1}{3}\delta+\frac{2}{3}\epsilon+2\eta). \tag{31}
$$

In this case U_ν becomes

$$
U_\nu \simeq \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\left(1+\frac{2}{9}\frac{\delta}{\bar{r}}-\frac{2}{9}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}-\frac{2}{9}\frac{\epsilon}{\bar{r}}\right) \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}}\left(1+\frac{2}{9}\frac{\delta}{\bar{r}}-\frac{2}{9}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}-\frac{2}{9}\frac{\epsilon}{\bar{r}}\right) \\
0 & -\sqrt{\frac{2}{3}}\left(1-\frac{1}{9}\frac{\delta}{\bar{r}}+\frac{1}{9}\frac{\epsilon}{\bar{r}}\right) & \frac{1}{\sqrt{3}}\left(1+\frac{2}{9}\frac{\delta}{\bar{r}}+\frac{4}{9}\frac{\epsilon}{\bar{r}}\right)
\end{pmatrix}, \tag{32}
$$

which induces the MNS matrix as

$$V_{MNS} = L^\dagger F^T U_\nu = \begin{pmatrix} 1 & L_{21} & -\frac{\sqrt{2}}{9} L_{21}\left(\frac{\delta}{\bar{r}} + \frac{\epsilon}{\bar{r}}\right) \\ L_{12} & 1 + L_{32}\left(\frac{\delta}{\bar{r}} - \frac{\epsilon}{\bar{r}}\right) & -\frac{\sqrt{2}}{9}\left(\frac{\delta}{\bar{r}} - 2\frac{\epsilon}{\bar{r}}\right) + L_{32} \\ L_{13} & \frac{\sqrt{2}}{9}\left(\frac{\delta}{\bar{r}} - \frac{\epsilon}{\bar{r}}\right) + L_{23} & 1 - \frac{\sqrt{2}}{9}\left(\frac{\delta}{\bar{r}} - \frac{\epsilon}{\bar{r}}\right) \end{pmatrix}. \tag{33}$$

This suggests that the mixing angles are given by

$$\sin^2 2\theta_{12} = 4L_{21}^2, \quad \sin^2 2\theta_{13} = \frac{2}{81} L_{21}^2 \left(\frac{\delta}{\bar{r}} + \frac{\epsilon}{\bar{r}}\right)^2, \quad \sin^2 2\theta_{23} = \frac{8}{81}\left(\frac{\delta}{\bar{r}} + 2\frac{\epsilon}{\bar{r}}\right)^2, \tag{34}$$

which means that the large mixing of the atmospheric neutrino solution is destroyed in the region of $1 \gg |\bar{r}| \gg \delta \gg \epsilon$. It is because the condition of $|\bar{r}| \gg \delta, \epsilon$ induces $U_\nu \simeq F$, which is just the case of Eq.(7).

The conclusion in case (ii) is that sufficient condition of $\tan\beta \leq 10$ must be satisfied for the MSW-S solution.

The democratic type of mass matrix texture is one of the most interesting candidate of quark and lepton mass matrices, which has been said to be able to induce the suitable solutions of the atmospheric and the solar neutrino problems. In this paper, we investigate whether the lepton flavor mixing angles in the democratic type of mass matrix are stable against quantum corrections or not in the minimal supersymmetric standard model with the dimension five operator which induces the neutrino Majorana mass matrix. We take the simple breaking patterns of $S_{3L} \times S_{3R}$ or $O(3)_L \times O(3)_R$ symmetries, and the scale where democratic textures are induced as $O(10^{13})$ GeV. Under the above conditions, we find that the stability of mixing angles in the democratic type of mass matrix against quantum corrections depends on the solar neutrino solutions. The maximal mixing of the VO solution is spoiled by in the experimental allowed region of $\tan\beta$. The MSW-L solution is spoiled by the quantum corrections in the region of $\tan\beta > 10$. On the other hand, the condition of $\tan\beta \leq 10$ is needed in order to obtain the suitable mass squared difference of the MSW-S solution. These strong constraints must be regarded for the model building of the democratic type of mass matrix. If we take m_h as the GUT scale of $O(10^{16})$ GeV, the constraints for the stability of the mixing angles against quantum corrections become more severe, that is, the MSW-L solution needs $\tan\beta < 8$, and the MSW-S solution needs $\tan\beta \leq 8$.

Finally, we should give a comment about flavor symmetry breaking patterns. We must notice that the assumption of $L_{ij} \ll 1$ $(i \neq j)$ plays the crucial role for our conclusions. If off-diagonal elements of L are of order one, our conclusions can be changed. For example, if L has the large mixing between the second and the third generations, $\sin^2 2\theta_{23} \simeq 8/9$ is realized with $r \gg 1$. Where the mixing angle of θ_{23} induced from the neutrino democratic mass matrix is not changed drastically by the quantum corrections, since the neutrino mass eigen-values have large hierarchies, just as the case of Type A in Ref.[13]. However, simple patterns of flavor symmetry breaking can not realize the large mixing between the second and the third generations in L.

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